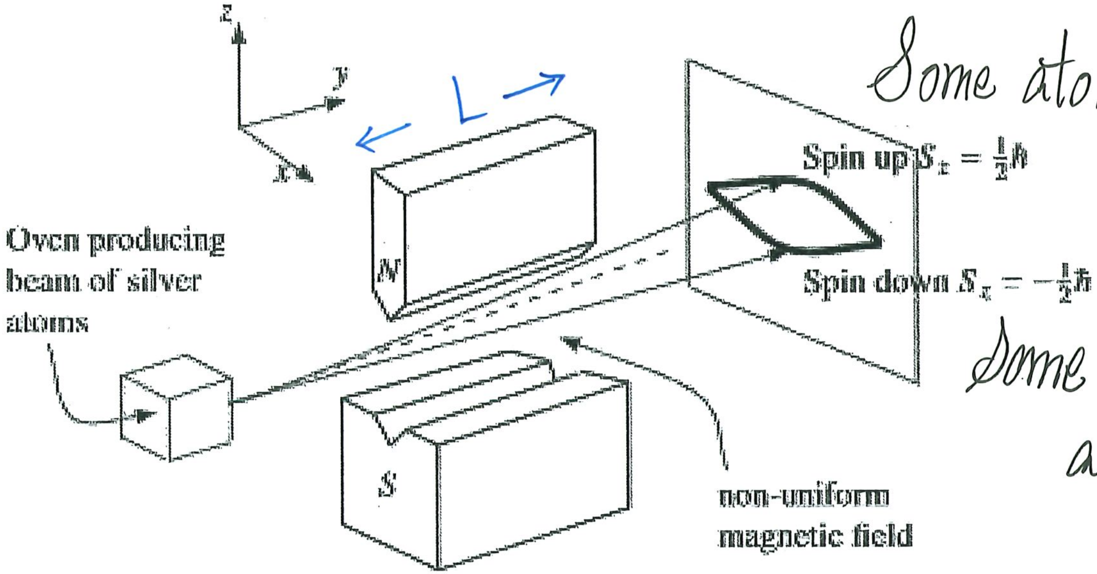
C. The Stern-Gerlach Experiment

- 1922 using Ag atoms (1 outer-electron in s-orbital) ($l=0$)
- 1927 (Phipps and Taylor) using hydrogen atoms (1s) ($l=0$)
- Pass beam of atoms through Stern-Gerlach set up
- Orbital AM ($l=0$), $(\mu_L)_z = 0 \Rightarrow$ No effect from orbital AM
- Thus, did <u>not</u> expect splitted beams even $\vec{B} \neq 0$ (with $\frac{\partial B}{\partial z}$)
- Moreover, any effect due to orbital AM gives odd number of splitted beams (e.g. $l=1$, 3 beams)
- But this was <u>NOT</u> what Stern and Gerlach observed!

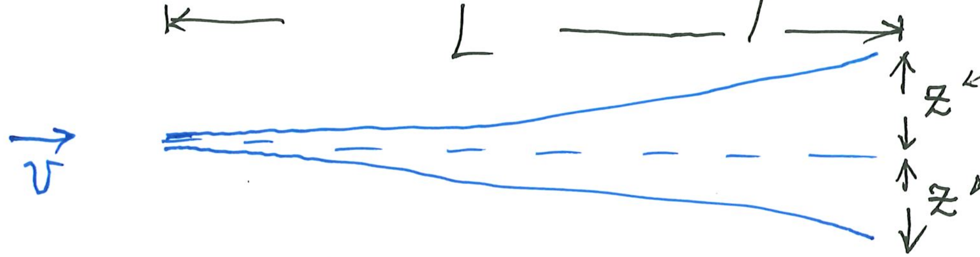
Observation



Some atoms deflected upward by a certain extent

Some atoms deflected downward by a certain extent

View result sideway



give μ_z information

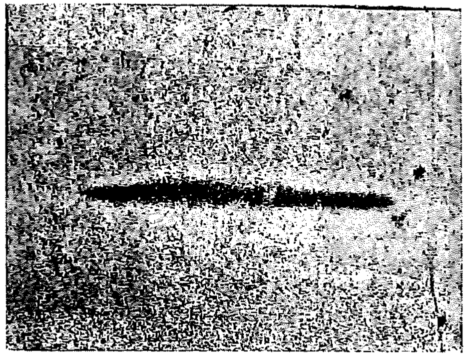
$$z = \frac{1}{2} a t^2 = \frac{1}{2} \frac{\text{Force}}{m} \left(\frac{L}{v}\right)^2 = \mu_z \cdot \frac{1}{4\underbrace{(KE)}_{\frac{1}{2}mv^2}} L^2 \left(\frac{\partial B}{\partial z}\right)$$

(kinematics)

Classic results of Stern[+] & Gerlach (1922)

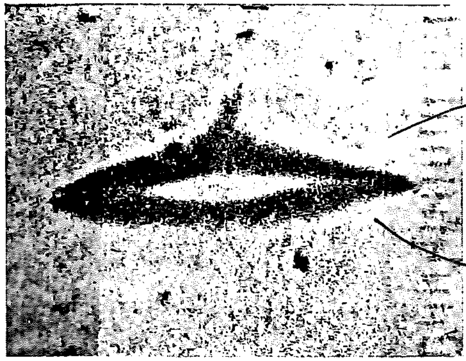
$\vec{B} = 0$

(a)



Turn On $\vec{B}$

(b)



One Up
[Two beams]
One down

[Not odd number]

The results of the Stern-Gerlach experiment. (a) The image of the slit with the field turned off. (b) With the field on, two images of the slit appear on the screen. The scale at right represents 1 mm.

Here comes Spin Angular Momentum

[Recall: $l = 0$]

Interpretation

- $\vec{\mu}$ due to something other than orbital AM
- μ_z can take on only two possible values

$\Rightarrow m_j = 1/2, -1/2$ only

$\therefore j = 1/2$ only (one value)

+ Stern was awarded the Nobel Prize in 1943, after 82 nominations since 1925! Gerlach was not awarded, probably due to political reason.

Experimental Facts

- Atoms come out as two split beams $\begin{cases} \text{some atoms feel an upward force} \\ \text{some atoms feel a downward force} \end{cases}$
- But orbital AM plays no role ($\because \ell = 0$ (s-orbital))
- Yet, atoms seem to carry a magnetic dipole moment with two eigenvalues for μ_z (z-component) ($\because$ 2 beams)
- This $\vec{\mu}$ does not come from $\vec{L}$
- With the idea that $\vec{\mu}$ is associated with an angular momentum (and it stems from the electron), there must be another AM that comes from the electron – the spin angular momentum of electron.

- Just like general AM, the same results (two beams come out) show up in any "direction" you place the Stern-Gerlach set up. [No special direction]
- Any direction, always see two beams (two $m_j = \frac{1}{2}, -\frac{1}{2}$ values)
- Consistent with: Can know J^2 and one component (called J_z)

D. Electron has an intrinsic Spin Angular Momentum

- Stern-Gerlach exp't: A new angular momentum due to electron
- Intrinsic property of electron: (Meaning - Every electron has this property)

  [c.f.: charge $(-e)$ of electron, mass m_e of electron]
- Goudsmit and Uhlenbeck (1925) [Pauli: "an additional quantum number"]

This is electron's "Spin"

$$\boxed{\begin{array}{l}\text{Every electron has a spin angular momentum } \vec{S} \\ \bullet\ \hat{S}^2 \text{ has only one eigenvalue } \frac{3}{4}\hbar^2 = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2 \\ \bullet\ \hat{S}_z \text{ takes on only } +\frac{1}{2}\hbar \text{ and } -\frac{1}{2}\hbar\end{array}} \quad (9)$$

Putting it in standard Angular momentum form:

(10)

$\hat{S}^2$ has one eigenvalue† $s(s+1)\hbar^2$

with s always takes on ½ for an electron

With $s = 1/2$, $m_s = +1/2, -1/2$ where $m_s\hbar$ are eigenvalues of $\hat{S}_z$

This "$s = 1/2$ only" leads to the description of

"Electron is a $\underbrace{\text{spin-}1/2}_{\text{magnitude } |S| = \sqrt{\frac{3}{4}}\hbar \text{ actually}}$ (spin-half) particle"

† Because Spin AM is very important, it has its own notation "S" instead of the general "J", its own quantum numbers s, m_s instead of j, m_j

Spin: an angular momentum Electron: charge $(-e)$

Expected a Spin Magnetic Dipole Moment $\vec{\mu}_s$ associated with Spin $\vec{S}$

Experimental Result [Stern-Gerlach type exp'ts]

Spin $\boxed{\vec{\mu}_s = \frac{-e}{m_e}\vec{S}}$ for electron c.f. $\boxed{\vec{\mu}_L = \frac{-e}{2m_e}\vec{L}}$ Orbital

off by a factor of 2

$$|\vec{\mu}_s| = \frac{e}{m_e}|\vec{S}| = \frac{e\hbar}{m_e}\sqrt{\frac{3}{4}} = \frac{e\hbar}{2m_e}\sqrt{3} = \sqrt{3}\,\mu_B$$ (always, intrinsic electron property)

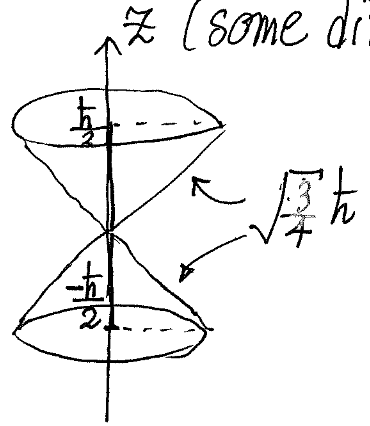
$$\boxed{\underbrace{\mu_{s,z}}_{z\text{-component}} = \frac{-e}{m_e}\underbrace{S_z}_{+\frac{\hbar}{2}\text{ or }-\frac{\hbar}{2}} \quad \text{takes on either } \mu_{s,z} = \underbrace{-\mu_B}_{\substack{(m_s=+1/2)\\(S_z=\hbar/2)}} \text{ or } \underbrace{+\mu_B}_{\substack{(m_s=-1/2)\\(S_z=-\hbar/2)}}}$$

Recall: 2 beams out in SG exp'ts

Important Remarks

- Electron's Spin is a property of electron
  [In contrast, electron's orbital AM is NOT an intrinsic property.
  For electron in 1s (ψ_{100}), its $|L| = 0$, $L_z = 0$
  For electron in 2p (say ψ_{210}), its $|L| = \sqrt{2}\hbar$, $L_z = 0$
  $\therefore$ Orbital AM depends on the atomic state it is in.]
- Spin AM eigenstates cannot be expressed as a wavefunction of space (x, y, z)
  [Electron's Spin is unrelated to electron's prob. density and thus $\psi(x,y,z)$, because it is intrinsic]
- A spinning electron picture is just a model

- Can also use a vector model as <u>a picture</u> of spin



- Don't mistaken that it is the electron[+] spinning (No! No! No!)

-  spin is a vector (precession) [picture]

It is just a model.

The results come from the <u>commutators</u>.

[+] The electron, for particle physics expts up to now, remains an elementary particle (no internal structure) no bigger than 10^{-18} m.

- Existence of spin AM is something beyond the Schrödinger Equation
  [does not come from solving TISE for atoms]

- Dirac (1927): Dirac's theory of an electron

  Dirac Equation for an electron [relativistic quantum mechanics]

  gives electron's spin in its solutions

  $\therefore$ **Spin angular Momentum is a relativistic effect**

No wonder Schrödinger Equation could not give it

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x}) \qquad \frac{p^2}{2m} = \frac{1}{2}mv^2$$

Non-relativistic starting point

- Since exp'tal results are consistent with general QM Angular momentum results, the spin AM operators must follow

$$[\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z, \quad [\hat{S}_y, \hat{S}_z] = i\hbar \hat{S}_x, \quad [\hat{S}_z, \hat{S}_x] = i\hbar \hat{S}_y$$

$$\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$$

$$[\hat{S}^2, \hat{S}_x] = [\hat{S}^2, \hat{S}_y] = [\hat{S}^2, \hat{S}_z] = 0$$

with exp'tal results indicating $s = 1/2$ only for electrons.

- Using $|s, m_s\rangle$ for $|j, m_j\rangle$ for $\hat{S}^2$ and $\hat{S}_z$

  Electron spin only allows

  $$|\tfrac{1}{2}, \tfrac{1}{2}\rangle \quad \text{and} \quad |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \qquad (11) \qquad \text{(formally)}$$

  always $s = \frac{1}{2}$ always $s = \frac{1}{2}$

- Boring to keep on writing the first "½" (always "½"),

  <u>short hand</u>: $|m_s = \tfrac{1}{2}\rangle \quad , \quad |m_s = -\tfrac{1}{2}\rangle \qquad (12)$

  OR $\quad |\tfrac{1}{2}\rangle \quad , \quad |-\tfrac{1}{2}\rangle \qquad (13)$

  OR $\quad |\uparrow\rangle \quad , \quad |\downarrow\rangle \qquad (14)$ (Quantum Information)

  OR $\quad \alpha \quad , \quad \beta \qquad (15)$ (Quantum Chemistry)